

A (short) introduction to the theory of weighted automata

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In the last twenty years, quantitative models and quantitative analysis have become of common usage in many areas of computer science, from software verification to natural language processing to machine learning for instance. Classical models of computation where sequences are either accepted or not are replaced by models where sequences are associated with values, be it probabilities, or costs, or outputs, in order to get more information on the computations and to enrich the models.

Finite automata is the simplest model of computation. Finite automata with weights is also the simplest computation model that incorporates multiplicities. But the variety of possible weights makes the subject wide open and rich of many developments.

In these four lectures, in conjunction with the two talks at the algebra seminar, I shall present weighted automata in the unifying language of linear algebra and with a point of view that is more common to classical control theory. In this framework, I set up the distinction between rationality and recognizability, and develop notions such as conjugacy and morphisms of automata and reduction of representations.

Tentative schedule:

Lecture 1 The computation model. The semiring of power series.
Rationality. The Fundamental Theorem of Finite Automata

Lecture 2 Representations and recognizability.
Kleene-Schützenberger Theorem.

Lecture 3 Conjugacy and morphisms of weighted automata. Bisimulation.

Lecture 4 Reduction of representations in the division ring case.
Decidability of equivalence.

Bibliography:

- (1) J. Sakarovitch, *Elements of Automata Theory*, Cambridge University Press, 2009 (Corrected English translation of *Éléments de théorie des automates*, Vuibert, 2003).
- (2) M. Droste *et al.*, eds, *Handbook of Weighted Automata*, Springer, 2009 (especially, Chap.4: Rational and recognisable power series).
- (3) J. Berstel and Ch. Reutenauer, *Noncommutative Rational Series with Applications*, Cambridge University Press, 2011.